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Twin surrogates to test for complex synchronisation

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Abstract. – We present an approach to generate (multivariate) twin surrogates (TS) based on recurrence properties. This technique generates surrogates which correspond to an independent copy of the underlying system, *i.e.* they induce a trajectory of the underlying system starting at different initial conditions. We show that these surrogates are well suited to test for complex synchronisation and exemplify this for the paradigmatic system of Rössler oscillators. The proposed test enables to assess the statistical relevance of a synchronisation analysis from passive experiments which are typical in natural systems.

Introduction. – The concepts of complex synchronisation and especially phase synchronisation (PS) have been intensively studied in recent years [1]. Indications of PS have been found in many laboratory and natural systems [2]. The corresponding studies are usually based on the computation of a measure which quantifies dependencies of the instantaneous phases of the time series. However, even though these measures may be normalised between 0 and 1, experimental time series often yield values which are neither close to 0 nor to 1 and hence are difficult to interpret. This problem can be overcome if the coupling strength between the two systems can be varied systematically and a rather large change in the measure can be observed ("active experiment"), as in laser experiments [1]. On the other hand, PS in natural systems, *e.g.* during epileptic seizures or between the heart beats of a mother with the ones of her foetus [3], frequently evades such an experimental manipulation ("passive experiment"), so that a hypothesis test should be performed [4].

In some cases, this problem has been tackled by interchanging the pairs of oscillators [3], *e.g.* the heart beats of other pregnant women were used as "natural surrogates". These surrogates are independent and hence not in PS with the original system. Hence, if the synchronisation index obtained for the original data is not significantly higher than the index obtained for the natural surrogates, there is not sufficient evidence to claim synchronisation. But even this approach has some drawbacks. The natural variability and also the frequency of the heart beats of the surrogate "mothers" are usually slightly different from the ones of the biological mother. Furthermore, the data acquisition can be expensive and at least in some cases problematic (*e.g.* in some states of the pregnancy).

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In these cases, we propose to test hypotheses on the basis of surrogates generated by a mathematical algorithm. Several approaches in this direction have been published. Usually, these are linear surrogates based on randomisation of the Fourier phases (*e.g.* the iterative Amplitude Adjusted Fourier Transform (iAAFT) surrogates) or wavelet-based surrogates [5]. They mimic the probability distribution, the individual spectra of both components of the original bivariate series as well as their cross-spectrum, *i.e.* their linear properties, but not the higher-order moments. In this case, the corresponding null hypothesis is that the putative synchronisation in the underlying system can be explained by a bivariate linear stochastic process observed through a nonlinear measurement function. The statistical specificity —considered as a count of false positives [6]— of this test is not always satisfactory, because the concept of PS assumes the mutual adaption of self-sustained oscillators, *i.e.* nonlinear deterministic systems. Furthermore, another innovative approach, the pseudo-periodic surrogates (PPS), has been proposed to test the null hypothesis that an observed time series is consistent with an (uncorrelated) noise-driven periodic orbit [7]. The PPS are closer to the surrogates needed to test for PS as they correspond to a trajectory of a deterministic system with noise. However, they are still not appropriate to test for PS, because they are not capable of mimicking chaotic oscillators. Surrogates based on a time shifting algorithm have also been studied, but the problem of truncating the time series or alternatively joining different blocks is still largely unsolved [8].

In this paper we present a new block-bootstrap–like concept for the generation of surrogates, which is based on the recurrences of a system. These surrogates mimic the dynamical behaviour of the system, *i.e.* not only the linear properties but also the nonlinear ones are preserved. In the case of two coupled oscillators A and B, the surrogates correspond to an independent copy of the joint system A and B, and hence, one surrogate of the (sub-)system A is not in PS with the observed (sub-)system B, and vice versa. The corresponding null hypothesis is that the putative synchronisation of the underlying system can be explained by an independent copy of the same system, *i.e.* the same system but just starting at different initial conditions. This idea is analogous to using (mathematical) "surrogates mothers" to detect mother-foetus heartbeat synchronisation.

In the case of deterministic systems, our surrogates correspond to a trajectory of the underlying system starting at different (random) initial conditions. Due to the random element in both situations, we will refer to the surrogates as alternative realizations of the respective system. The main idea is to find points of the measured trajectory which are not only neighbours, but also share the same neighbourhood in phase space (*twins*), *i.e.* all other points are either neighbours of both or of neither of them. Once the twins of the trajectory have been localised, new surrogate trajectories are generated by substituting randomly the next step in the trajectory by either its own future or the one of its twin.

Algorithm to generate twin surrogates. – The algorithm to generate the surrogates is based on the recurrence matrix

$$R_{i,j} = \Theta\left(\delta - ||\vec{x}(i) - \vec{x}(j)||\right), \qquad i,j = 1,\ldots,N, \tag{1}$$

where $\Theta(\cdot)$ denotes the Heaviside function, $||\cdot||$ the maximum norm and δ is a predefined threshold. $\vec{x}(i)$ denotes the vector of the trajectory of the system in phase space at time $t = i\Delta t$, with Δt being the sampling time of the trajectory and $i = 1,\ldots,N$. In the case that only a scalar time series has been observed, the trajectory of the system has to be reconstructed using some embedding technique. Coding the "1's" in the matrix as black dots and the "0's" as white ones, we obtain the recurrence plot (RP) [9]. It is important to note that the RP contains all topological information about the underlying attractor, as suggested in [10]. Hence, a first idea for the generation of surrogates is to change the structures in an

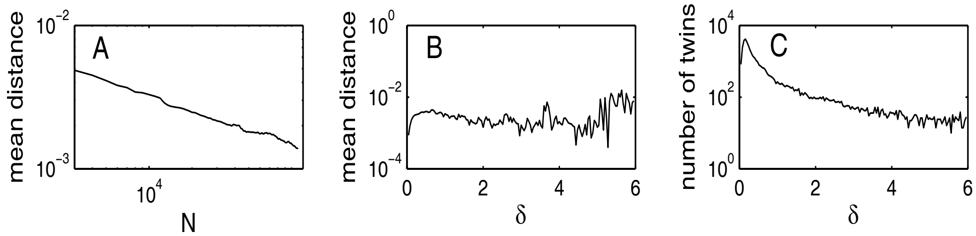

Fig. 1 – A) Mean distance of the jumps introduced when generating TS of the Rössler system in dependence on the length N of the time series (for $\delta = 0.25$). B) Mean distance of the jumps in dependence on the threshold δ (for $N = 10000$). The diameter of the attractor is about 32. A δ of 6 corresponds to a percentage or recurrent points in the recurrence matrix of about 20%. C) Number of twins in dependence on δ.

RP consistently with the ones produced by the underlying dynamical system (the structures in the RP are linked to dynamical invariants of the underlying system, such as the correlation entropy and the correlation dimension [10]) and then reconstruct the trajectory from the modified RP. Furthermore, we use the fact that in an RP there are identical columns, *i.e.* $R_{k,i} = R_{k,j}\ \forall k$. Thus, there are points which are not only neighbours (*i.e.* $||\vec{x}(i) - \vec{x}(j)|| < \delta$), but which also share the same neighbourhood. We call these points twins. Twins are special points of the time series as they are indistinguishable considering their neighbourhoods but in general different and hence, have different pasts and —more important— different futures. The key idea of how to introduce the randomness needed for the generation of surrogates of a deterministic system is that one can jump randomly to one of the possible futures of the existing twins. A surrogate trajectory $\vec{x}^s(i)$ of $\vec{x}(i)$ with $i = 1, \ldots, N$ is then generated as follows: i) Identify all pairs of twins, *i.e.* all pairs $\vec{x}(i)$ and $\vec{x}(j)$ such that $R_{i,k} = R_{j,k}$ for $k = 1, \ldots, N$. ii) Choose an arbitrary starting point $\vec{x}(l)$ and set $\vec{x}^s(1) = \vec{x}(l)$. iii) Next, we generate the twin surrogate iteratively. The j-th entry of the surrogate may be given by $\vec{x}^s(j) = \vec{x}(m)$. If $\vec{x}(m)$ has no twins, set $\vec{x}^s(j+1) = \vec{x}(m+1)$. If, on the other hand, $\vec{x}(n)$ is a twin of $\vec{x}(m)$, set $\vec{x}^s(j+1) = \vec{x}(m+1)$ or $\vec{x}^s(j+1) = \vec{x}(n+1)$ with equal probability(1). Step 3 is then iterated until the surrogate time series has the same length as the original one.

This algorithm creates twin surrogates (TS) which are shadowed by (typical) trajectories of the system in the limit of an infinitely long original trajectory. Note that the TS are multivariate surrogates, *i.e.* if the original trajectory is d-dimensional, the TS are also d-dimensional. Already for rather short trajectories, the *errors* or *jumps* $||\vec{x}(i) - \vec{x}(j)||$ introduced by the TS generation are rather small (i and j denote the time indices of two twins). Figure 1A shows how the mean distance of the jumps (normalised by the diameter of the attractor) changes in dependence on the length of the time series N for the Rössler system(2) with parameters $a = b = 0.2$ and $c = 5.7$. We observe that longer time series lead to smaller jumps.

The choice of the parameter δ. – The TS algorithm contains only one parameter: the threshold δ (eq. (1)). In order to study the influence of this parameter, we compute the mean distance of the jumps introduced in the generation of the TS in dependence on the threshold δ. Figure 1B shows this dependence for the Rössler system. We observe that for a relatively large range of values of δ (approximately from 0 to 5) the mean distance of the jumps between the twins fluctuates around a constant value. That means that the quality of the TS does

(1)If triplets or multiplets occur one proceeds analogously.
(2)The equations are $\dot{x} = -(y + z)$, $\dot{y} = x + ay$, $\dot{z} = b + z(x - c)$.

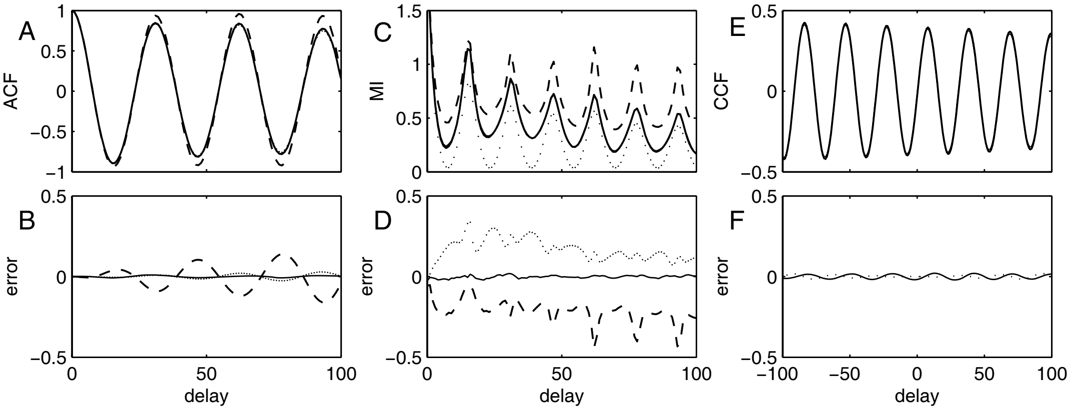

Fig. 2 – A) Autocorrelation function (ACF) of the x-component of the Rössler oscillator (bold), ACF of one iAAFT surrogate (dotted), ACF of one PPS (dashed) and ACF of one TS (solid). B) Difference between the ACF of the original time series and the ACF of the iAAFT surrogate (dotted), the ACF of the PPS (dashed) and the ACF of the TS (solid). C) and D) Analogously to A) and B) but for the mutual information (MI). E) Cross correlation function (CCF) between the x-components of two mutually coupled Rössler oscillators (bold) and for the x-components of bivariate iAAFT surrogates (dotted) and TS (solid). F) Difference between the CCF of the original time series and the CCF of the iAAFT surrogates (dotted) and the CCF of the TS (solid).

not depend crucially on the choice of δ. In continuous systems the number of twins may strongly depend on δ and on the distribution of the data (fig. 1C). However, as the number of statistically independent surrogates is usually much larger than the number of twins and as the jumps remain small, the choice of δ does practically not cause problems. A choice of δ corresponding to 5%–20% of black points in the RP is appropriate. Numerical simulations with other systems, such as the Lorenz system or the Bernoulli map, confirm these considerations.

Comparison to other kinds of surrogates. – Next we compare the TS to the PPS and iAAFT surrogates. Therefore, we compare the autocorrelation function (ACF) and the mutual information (MI) of a time series with the ones of the three different kinds of surrogates. In fig. 2A the ACF of the x-component of the Rössler system in chaotic regime with dynamical noise ($a = 0.398$, $b = 2$, $c = 4$ and Gaussian noise with a standard deviation of 0.05 added to each component at each step of the integration) (bold) is compared with the ACF of one iAAFT surrogate (dotted), of one PPS (dashed) and one TS (solid). In fig. 2B the difference between the ACFs of each surrogate and of the original time series is shown for a better comparison. The ACF of the original time series coincides with the ACF of the iAAFT surrogate and the one of the TS. This is as expected, because both the iAAFT surrogates and the TS mimic the linear properties of the underlying system. However, the ACF of the PPS does not coincide with the one of the original time series, because the PPS correspond to the null hypothesis of a noise-corrupted periodic system and hence the amplitude of the ACF does not decay as for a chaotic system [7].

Figure 2C displays the MI of the original time series (bold), the MI of the iAAFT surrogate (dotted), the MI of the PPS (dashed) and the MI of the TS (solid). Again, fig. 2D shows the difference between the MI of the original time series and the MI of each kind of surrogate. The only surrogate which reproduces correctly the MI of the original time series is the TS. The errors for the iAAFT surrogate and the PPS are rather large. This result is consistent with the respective null hypotheses of the three different surrogates techniques: the iAAFT surrogates are consistent with a Gaussian linear process observed through a nonlinear measurement function. Hence, only the linear statistics are preserved by the iAAFT surrogates. The PPS are consistent with a periodic system driven by noise. Therefore, the nonlinear statistics, such as the MI are not preserved by this kind of surrogates. Only the TS are consistent with exactly the same underlying system (but starting at different initial conditions) and hence, they preserve the linear and the nonlinear properties of the original time series. Numerical simulations with a variety of other systems, such as the Lorenz system, the logistic map and even simple stochastic processes confirm these results.

Test for phase synchronisation. – Now we exemplify a highly relevant application of the TS, namely how to use them to test for PS. The idea behind this approach is similar to the one by means of "natural surrogates" in the mother-foetus heartbeat synchronisation [3]. Consider two coupled self-sustained oscillators, $\vec{x}_1(t)$ and $\vec{x}_2(t)$. Then, we generate M surrogates of the joint system, *i.e.* $\vec{x}_1^{s_i}(t)$ and $\vec{x}_2^{s_i}(t)$, with $i = 1, \ldots, M$. These surrogates are independent copies of the system, *i.e.* trajectories of the system starting at different initial conditions. Note that the coupling between $\vec{x}_1(t)$ and $\vec{x}_2(t)$ is also mimicked by the TS, *e.g.* the cross correlation function (*e.g.* x-components) of the underlying oscillators is correctly reproduced by the TS (fig. 2E, F). The same holds for the cross mutual information.

Next, we compute the differences between the phases of the original system $\Delta\Phi(t) = \Phi_1(t) - \Phi_2(t)$, where $\Phi_{1,2}(t) = \arctan(y_{1,2}(t)/x_{1,2}(t))$ [1], and compare them with $\Delta\Phi^{s_i}(t) = \Phi_1(t) - \Phi_2^{s_i}(t)$, where $\Phi_2^{s_i}(t)$ denotes the phase of the surrogate i of the (sub-)system 2 (one could also consider $\Phi_1^{s_i}(t) - \Phi_2(t)$). Then, if $\Delta\Phi(t)$ does not differ significantly from $\Delta\Phi^{s_i}(t)$ with respect to some index for PS, the null hypothesis cannot be rejected and hence, we do not have enough evidence to state PS.

As a test case, we consider two non-identical, mutually coupled Rössler oscillators [1] with a frequency mismatch of $\nu = 0.015$(3). In this "active experiment", we vary the coupling strength ε from 0 to 0.08 and compute a PS index for the original trajectory for each value of ε. Next we generate 200 TS choosing $\delta = 1.0$(4) and compute the PS index between the original first oscillator and the surrogates of the second one. As PS index we use the mean resultant length R of complex phase vectors [11],

$$R = \left| \frac{1}{N} \sum_{t=1}^{N} \exp[i\Delta\Phi(t)] \right| . \tag{2}$$

It takes on values in the interval from 0 (non PS) to 1 (perfect PS) [11]. Let R^{s_i} denote the PS index between the observed first oscillator and the surrogate i of the second one. To reject the null hypothesis at a significance value α, R must be larger than $(1-\alpha)\cdot 100$ percent of all R^{s_i}. Note that this corresponds to computing the significance level α from the cumulative histogram of R^{s_i}. As the TS are supposed to correspond to independent copies of the system, we also perform the test with (perfect) equation based surrogates (EBS) —each starting at different initial conditions. Figure 3A shows the results for R of the original system (bold) and the 1% significance level based on the TS (solid), on the EBS (dashed) and on the iAAFT surrogates (dotted). Figure 3B displays the difference between R of the original system and the 1% significance level. To validate this result, we plot in fig. 3C the four largest

(3)The equations are $\dot{x}_{1,2} = -(1 \pm \nu)y_{1,2} - z_{1,2} + \varepsilon(x_{2,1} - x_{1,2})$, $\dot{y}_{1,2} = (1 \pm \nu)x_1 + 0.15y_{1,2}$, $\dot{z}_{1,2} = 0.2 + z_{1,2} + z_{1,2}(x_{1,2} - 10)$.

(4)This choice corresponds to ~ 1000 twins. The results for the test choosing $\delta = 3.0$ (~ 100 twins) and $\delta = 5.0$ (~ 10 twins) are up to statistical fluctuations the same.

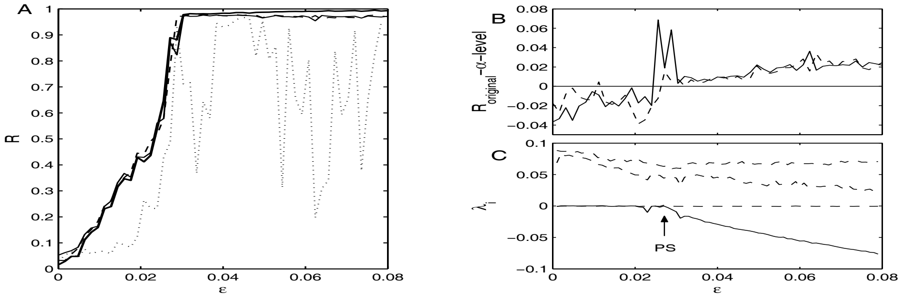

Fig. 3 – A) R of the original data (bold) and significance level of 1% based on the TS with $\delta = 1.0$ (solid), on EBS (dashed) and on the iAAFT surrogates (dotted). B) Difference between R of the original data and 1% significance level based on the TS (solid) and on the EBS (dashed). C) Four largest Lyapunov exponents for the 6-dimensional system considered. λ_4 is highlighted (solid) and the arrow indicates the transition to PS.

Lyapunov exponents λ_i of the joint system in dependence on the coupling strength ε. The onset of PS occurs when λ_4 becomes negative [1]. This occurs at $\varepsilon \sim 0.024$ (fig. 3C), which approximately coincides with the intersection of the curve of R for the original system and the significance level from the TS and the natural surrogates (zero-crossing of the curves in fig. 3B). For $\varepsilon < 0.024$ both oscillators are not in PS, and indeed R of the original system is not significantly larger than R^{s_i} of the surrogates, *i.e.* the null hypothesis is not rejected and hence we have not enough evidence to claim synchronisation. For $\varepsilon > 0.024$ both oscillators are in PS, and indeed R of the original system is significantly larger than R^{s_i} of the surrogates, *i.e.* the null hypothesis is rejected. In contrast to the iAAFT surrogates, the TS yield the same results as the (ideal) EBS. Furthermore, simulations have shown that the quality of the TS and the results of the test are the same if delay embedding is used.

Therefore, we recognise successfully the PS region by means of the TS. Note that also the significance limit increases at the transition to PS (fig. 3A). As the TS mimic both the linear and nonlinear properties of the system, the surrogates of the second oscillator have the same mean frequency as the first original oscillator in the PS region. Hence R^{s_i} is rather high. However, $\Phi_1(t)$ and $\Phi_2^{s_i}(t)$ do not adapt to each other, as they are independent. Hence, the value of R for the original system is significantly higher than the R^{s_i}. We state in conclusion, that even though the value for a normalised PS index is higher than 0.97 (right side of fig. 3A), this does not give conclusive evidence for PS. Hence, *the knowledge of the PS index alone does not provide sufficient evidence for PS*([5]).

Sensitivity and statistical specificity of the test. – Finally, we perform an analysis of the sensitivity and specificity of the test. For each of the following three cases we generate 100 trajectories each starting at different initial contitions for two Rössler systems: i) identical but non-coupled oscillators starting at different initial conditions, ii) two weakly and mutually coupled but not phase synchronised oscillators, and iii) two mutually coupled and phase synchronised oscillators. We then compute 200 TS for each of the 100 trajectories in each case, *i.e.* we perform 300 independent tests. In the case i) the null hypothesis is true, and at a significance level of $\alpha = 1\%$, it is erroneously rejected (type-I error) only in 1 out of the 100

([5])Note that the more phase coherent the oscillators are, the more difficult it is to decide whether they are in PS or not. A certain phase diffusion, which allows to measure the adaptation of the phases of the interacting oscillators, is necessary to detect PS. However, the test based on the TS reveals whether there is enough evidence for PS.

cases. This is a rather auspicious result, as due to the identical frequencies, it is extremely difficult to recognise that there is no PS in this case [12]. In the case ii) ($\varepsilon = 0.02$ and $\nu = 0.015$), the null hypothesis was also true and it was not erroneously rejected (type-I error) in any of the 100 tests. In the case iii) ($\varepsilon = 0.045$ and $\nu = 0.015$), the null hypothesis was not true, and in all 100 test runs the null hypothesis was correctly rejected (there were no type-II errors). These results indicate that the sensitivity and the specificity of the test are rather good.

Conclusions. – In conclusion, we propose a new method for generating surrogates based on the concept of recurrence. These TS correspond to an independent copy of the underlying system, *i.e.* to a trajectory of the system starting at different initial conditions. We have shown that the TS can be used to test for PS, as illustrated for the prototypical system of Rössler oscillators. The generation of TS and the proposed test for PS have been successfully applied also to further simulated and meausred systems. Hence, the proposed test makes it possible to assess the statistical relevance of a data analysis from passive experiments (which are typical in natural systems) with respect to PS.

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